ANALYTIC IMPRECISE-PROBABILISTIC STRUCTURAL RELIABILITY ANALYSIS

J. Sadeghi[1], **M. De Angelis**[1] **and E. Patelli**[1]
[1]: Institute for Risk and Uncertainty, University of Liverpool, United Kingdom.

J.C.Sadeghi@liverpool.ac.uk, mda@liverpool.ac.uk, edoardo.patelli@liverpool.ac.uk

ABSTRACT

In this paper analytic expressions are given to evaluate the reliability of systems consisting of components, connected in parallel or series, subject to probability box failure distributions. This development allows engineers to evaluate the structural reliability of systems without having to resort to Optimisation and Monte Carlo simulation, which are costly both in terms of the time spent programming and the time required for computation. The results obtained are also more accurate than the equivalent results calculated by simulation. We compare a worked example for a simple series system to the equivalent result from simulation.

1. INTRODUCTION

Probabilistic safety analysis (PSA) was first introduced in the 1970s as a means of establishing the probability of a certain amount of radiation release to the environment from a nuclear structure. It is perceived to address many of the weaknesses of deterministic analysis [1]. For example, deterministic analysis relies heavily on engineering conservatism which could be difficult to quantify in practice. In addition, it is not always clear what the most conservative value for a particular parameter is when performing a black box analysis.

Probabilistic safety analysis is broken down into three levels. Level 1 PSA studies the reactor and determines accident sequences which are likely to result in a release from the reactor pressure vessel (RPV). Level 2 considers the containment structure, and how likely this is to fail in an accident. This is done by creating a fragility curve for the containment, which quantifies the failure probability at a particular load. Level 3 PSA combines the information produced by level 1 and level 2 PSA to provide the probability of radiation release to the environment [2].

In recent years, techniques from the area of imprecise probability have been increasingly applied to Probabilistic Safety Analysis studies in academic literature [3] [4]. Imprecise probabilities offer a natural framework to model uncertainty due to lack of knowledge (epistemic uncertainty). Epistemic uncertainty is particularly important in the nuclear industry where there is often a lack of sufficient data to completely model relevant phenomena. This uncertainty can be modelled as interval uncertainty in the parameters of traditional probability distributions, which is known as probability bounds analysis, where the imprecise distributions themselves are referred to as probability boxes [5]. This approach strikes a pragmatic compromise between engineering conservatism and overly optimistic analyses. However, the techniques proposed usually require sophisticated simulation techniques [6]. For example, to propagate uncertainty through a complex black box model expensive Monte Carlo or optimisation methods are used [7]. Although the imprecise probabilistic methods offer robust analysis, their use in the nuclear industry is not yet widespread. In [8] recommendations are made for how available data can be used to define probability boxes. In [9] approximate results were derived for systems where the First Order Reliability Method (FORM) could be applied. In (conventional) structural probabilistic safety analysis often the relations used are simple analytic expressions which, in contrast to the methods based on imprecise probability, allow the failure probability of the system to be computed with no Monte Carlo simulation at all. This offers two significant advantages. Firstly, the computational time required to complete the calculations is greatly reduced, which allows projects to be completed on shorter timescales and less money to be spent on High Performance Computing (HPC). Secondly, the time of engineers is saved as they are not required to spend large amounts of time programming Monte Carlo simulations, which reduces expenditure for their employer, and consequently benefits the industry as a whole.

In the United States the nuclear regulator [10] refers to the work of Kennedy who provides many analytic relationships to establish the fragility curve for a containment with a conventional probabilistic treatment [11]. The effect of epistemic uncertainty in PSA with conventional probability was considered in [12]. In this paper we will propose imprecise probabilistic analogues to many of the probabilistic formulae proposed in Kennedy's paper which have become standard expressions used in probabilistic safety analysis. In this way we hope to unite the conventional literature which is applied to PSA in industry with relatively recent developments in imprecise probability. The analysis will make extensive use of the probability boxes introduced in probability bounds theory. We will demonstrate how to establish the fragility curve of a system when components are connected in parallel or series, and when the failures of the components may have unknown dependencies. We will also demonstrate how this can be used to calculate the failure probability when there is additional imprecision in the load distribution. Then we will demonstrate how to establish a probability box fragility curve when the product of two random variables must be considered. All of the above could be combined with an event tree to yield the expected radiation release to the environment.

The merit of this approach is that the entire fragility curve can be constructed by one analyst using a spreadsheet package, without the requirement to use HPC resources or complicated simulation techniques which would require large amounts of time spent programming by the analyst. Therefore the benefits of traditional PSA approaches are retained whilst also obtaining the advantages of using probability bounds theory. Our results correspond to a specific, precise version of the equations presented in [9]. Our results are related to those found in [13].

In Section 2. an brief overview is presented of a typical PSA calculation used to determine the fragility curve of a system. In Section 3. we propose analogues to the expressions from Section 2. using probability bounds analysis. In Section 4. a simple example is presented. In Section 5. a brief summary is given.

2. Probabilistic Safety Analysis

In PSA level 2 the main goal is to establish the fragility curve of the nuclear structure. In seismic hazard analysis the fragility curve expresses the failure probability of the structure as a function of the peak ground acceleration. This can then be used to conduct safety analysis once the conditions inside the reactor (the 'source term') and the external conditions are known [14].

For a system, S, of components, c_i, connected in series (i.e. the system will fail if one component fails) the fragility of the system, $f(s|a)$, at a damage measure a is given by

$$f(s|a) = 1 - \prod_{c_i \subset S} [1 - f(c_i|a)], \tag{1}$$

when the fragilities of the individual components are independently distributed [11]. If the dependence is not known then the value of $f(s|a)$ given by Eqn. 1 is an upper bound which, for the small probabilities relevant to this type of analysis [15], approaches the value given by the more general Boole's inequality (due to the rare event approximation), which is itself equal to the right hand side of the Fréchet inequality [16] for the upper bound of the union of n events:

$$\max(P(A_1), P(A_2), ..., P(A_n)) \leq P\left(\bigcup_i A_i\right) \leq \min(1, P(A_1) + P(A_2) + ... + P(A_n)), \tag{2}$$

the other Fréchet inequality (which applies for components connected in parallel) being

$$\max(0, P(A_1) + P(A_2) + ... + P(A_n) - (n-1)) \leq P\left(\bigcap_i A_i\right) \leq \min(P(A_1), P(A_2), ..., P(A_n)). \tag{3}$$

Note that both Boole's inequality and the Fréchet inequalities are conservative bounds which should be used when the dependence between failure events is unknown. If the components are connected in parallel (i.e. the system has redundancy and fails if every component fails) then the fragility is given by

$$f(s|a) = \prod_{c_i \subset S} [f(c_i|a)], \tag{4}$$

if their fragilities are independently distributed. If the dependence is not known then the value of $f(s|a)$ given by Eqn. 4 is an upper bound [11], which is equivalent to the Fréchet inequality (Eqn. 3). These formulae can also be applied to connected systems which form super systems, in which case the unknown dependence versions on the equations should be used [11].
In this analysis $f(c_i|a)$ is usually modelled as a log normally distributed random variable, because the physical quantities being modelled must be greater than zero, i.e.

$$f(c_i|a) = \phi\left(\frac{log(\frac{a}{\beta})}{\sigma}\right), \tag{5}$$

where β represents the median failure value and σ is the standard deviation, and ϕ is the standard normal CDF. In fact, the value used for σ is typically obtained for the 'composite' distribution which is an averaged distribution obtained by combining the aleatory (i.e. the true uncertainty, σ_a) and the epistemic uncertainty (our uncertainty in the distribution parameters, σ_e) [17][18]. This applies when the uncertainty in β is given by a lognormally distributed random variable with parameters $\bar{\beta}$ and σ_e. For the composite distribution, σ is the euclidean norm of the two lognormal standard deviations, i.e. $\sigma = \sqrt{\sigma_a^2 + \sigma_e^2}$ and $\beta = \bar{\beta}$. This distribution is assumed to be conservative, since it approaches the asymptotic values in the tails of the distribution were we to consider the set of distributions described by the assumed epistemic distribution in full [11].

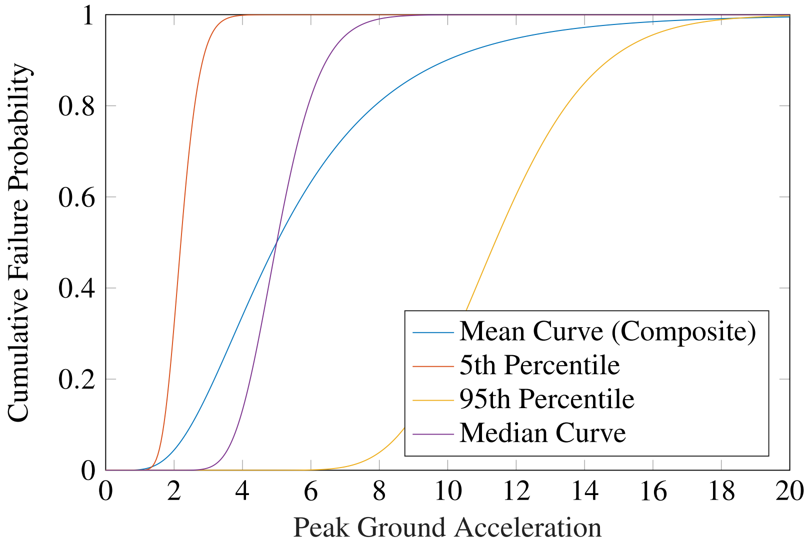

Figure 1 – The composite curve compared to the mean curve ($\beta = \bar{\beta}$ and $\sigma = \sigma_a$, and the curves with 5th and 95th percentiles of β and $\sigma = \sigma_a$. In the example $\sigma_a = 0.2$, $\bar{\beta} = 5$ and $\sigma_e = 0.5$.)

However, in many cases there may be insufficient data to truly know that our epistemic uncertainty is log-normally distributed. Figure 1 shows an example of a composite distribution compared to the median fragility curve and the 5th and 95th percentiles of the epistemic uncertainty.

3. Probability bounds analysis

3.1 Fragility Curve

Let us consider the fragility distribution for a general component given by

$$f(c_i|a) = \phi\left(\frac{log(\frac{a}{\beta_a})}{\sigma_a}\right). \tag{6}$$

Instead of considering β_a as a random variable and finding the composite distribution we will instead consider uncertainty in β_a and σ_a as intervals. This enables the random variables to be converted into probability boxes, since probability boxes are nothing more than cumulative distribution functions with interval imprecision on the distribution parameters. This framework is attractive for several reasons. Firstly we do not need to assume a distribution for our epistemic uncertainty, which permits a robust analysis even with limited data. Secondly, instead of having to find the composite distribution we can simply find the envelope of our distributions.
If $\beta_a = [\underline{\beta_a}, \bar{\beta_a}]$ and $\sigma_a = [\underline{\sigma_a}, \bar{\sigma}_a]$ then the distributional probability box can be converted to a

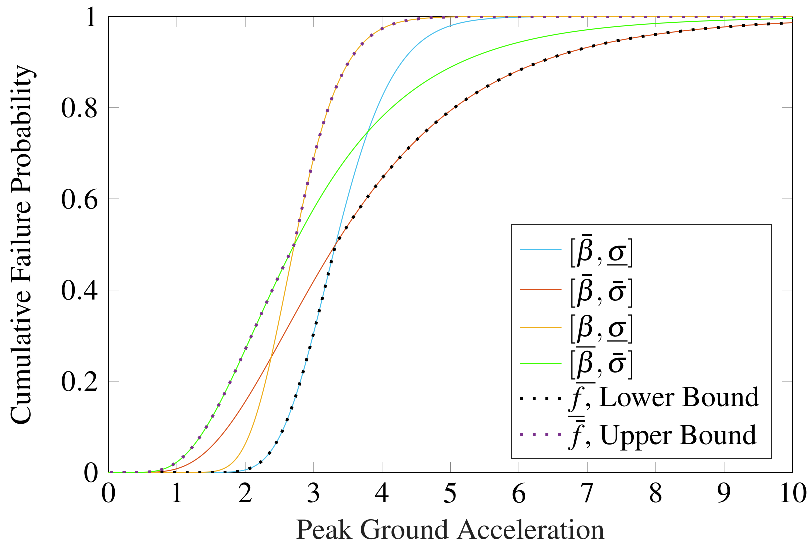

Figure 2 – A comparison of extreme fragility curves enclosed within the fragility probability box.

distribution free probability box where the upper bound of the fragility is given by

$$\bar{f}(c_i|a) = \phi\left(\frac{log(\frac{a}{\underline{\beta}_a}) - |log(\frac{a}{\underline{\beta}_a})|}{2\bar{\sigma}_a}\right) + \phi\left(\frac{log(\frac{a}{\underline{\beta}_a}) + |log(\frac{a}{\underline{\beta}_a})|}{2\underline{\sigma}_a}\right) - 0.5, \tag{7}$$

and the lower bound of the fragility is given by

$$\underline{f}(c_i|a) = \phi\left(\frac{log(\frac{a}{\bar{\beta}_a}) + |log(\frac{a}{\bar{\beta}_a})|}{2\bar{\sigma}_a}\right) + \phi\left(\frac{log(\frac{a}{\bar{\beta}_a}) - |log(\frac{a}{\bar{\beta}_a})|}{2\underline{\sigma}_a}\right) - 0.5, \tag{8}$$

where the modulus operator has been used to create the discontinuity in the envelope at $x = \bar{\beta}_a$ and $x = \underline{\beta}_a$. These bounds are shown in Figure 2.

For systems containing components in series or parallel the fragility can be calculated by using Eqn. 1. Alternatively, if the failure dependence is unknown we can use the relevant Fréchet inequality (Eqns. 2 and 3) to yield the fragility. Alternatively, the rare event approximation can be used in the tails.

Therefore, using Eqn. 3 with Eqn. 7 and Eqn. 8, for components in series, the probability of failure at a particular ground motion, a, with certainty falls in the interval given by

$$f(s|a) \in \left[\max_i \left(\phi \left(\frac{log(\frac{a}{\bar{\beta}_{ai}}) + |log(\frac{a}{\bar{\beta}_{ai}})|}{2\bar{\sigma}_{ai}} \right) + \phi \left(\frac{log(\frac{a}{\bar{\beta}_{ai}}) - |log(\frac{a}{\bar{\beta}_{ai}})|}{2\underline{\sigma_{ai}}} \right) - 0.5 \right), \right.$$
$$\left. \sum_i \left(\phi \left(\frac{log(\frac{a}{\underline{\beta_{ai}}}) - |log(\frac{a}{\underline{\beta_{ai}}})|}{2\bar{\sigma}_{ai}} \right) + \phi \left(\frac{log(\frac{a}{\underline{\beta_{ai}}}) + |log(\frac{a}{\underline{\beta_{ai}}})|}{2\underline{\sigma_{ai}}} \right) - 0.5 \right) \right]. \quad (9)$$

3.2 Failure Probability

For a general load distribution the failure probability, which can be calculated directly by this integral or by the ‘difference distribution approach’ [14], is given by

$$P_f = \int_0^\infty \frac{dH}{da} f(s|a) da, \quad (10)$$

where H is the ‘load’ CDF. In general this integral is not solvable analytically. When $H(a)$ and $f(s|a)$ are log normally distributed it is simple to calculate the distribution of P_f, since one can use the ratio method (i.e. define the failure region by $\frac{H(a)}{f(s|a)} \geq 1$), which using the transformation $P_f = log(w)$ allows the problem to be posed as finding the distribution of the difference between two normally distributed variables, which has a well known solution [19]. This integral cannot be solved analytically when the fragility curve is the envelope of a lognormal probability box due the discontinuity of the function, and therefore numerical integration would be necessary.
It is however possible to find an analytic expression for $\bar{P}_f$ by considering the distributional probability box, since we can use the result for the case when the fragility is given by a precise CDF, combined with the Fréchet bounds and interval arithmetic.
Let $\frac{dH}{da}$ be lognormally distributed with β_l and σ_l. Let L represent the load distributed with $\frac{dH}{da}$ and F represent the strength with CDF $\underline{f}(s|a)$, then

$$\bar{P}_f = P(\frac{L}{F} \geq 1) = P(log(\frac{L}{F}) \geq 0) = P(log(L) \geq log(F)), \quad (11)$$

where clearly $log(L)$ is normally distributed. In this case the failure probability (which is found by performing the well known difference distribution integral [19]) can be calculated as

$$P_f = \phi \left(-\frac{\log \beta_S - \log \beta_l}{\sqrt{\sigma_S^2 + \sigma_l^2}} \right), \quad (12)$$

for a system with load distribution l and strength distribution S. To calculate an upper bound on the failure probability for a series system we evaluate the maximum and minimum of Eqn. 10 with $\vec{\mu} \in [\underline{\vec{\mu}}, \bar{\vec{\mu}}], \vec{\sigma} \in [\underline{\vec{\sigma}}, \bar{\vec{\sigma}}]$ and Eqn 2. Analogously, for components in parallel a similar result can be obtained from Eqn. 3. For simple systems these bounds provide useful analytic quantification of the reliability of the system under epistemic uncertainty. However, for more complex systems numerical integration may be necessary.

It is likely that there is uncertainty in β_l and σ_l. If this is the case then the analysis can be made robust using an uncertainty quantification approach for the load distribution which is analogous to the approach used for the fragility.

In some works, such as ASCE 43-05 [20], the hazard curve has been modelled as a power law, since this is a good approximation to the Cauchy-Pareto complementary cumulative distribution function [21]. Such an equation takes the form of

$$H(a) = 1 - k_1 a^{-K_H}, \tag{13}$$

where k_1 and K_H are positive fitted constants. With a lognormal fragility in the parametrisation used in this paper, the failure probability for a single component is given by

$$P_f = H(\beta)\exp\left(K_H\sigma\right)^2/2. \tag{14}$$

When there is interval imprecision in K_H, k_l and σ we can obtain bounds on the failure probability, and this result can be generalised trivially to the case of a parallel or series system using the formulae given in Section 2. In order to facilitate the efficient use of this paper, all of the results obtained in this section are summarised in Table 1.

3.3 Product of lognormally distributed random variables

Often the fragility curve for a component must be established by considering the product of a number of random variables with lognormal distributions. If this is the case then the probability bounds analysis approach can be extended to allow us to find the relevant fragility curve. To demonstrate, consider a general random variable d which is given by the product of other random variables, i.e. $d = q\frac{a^r b^s}{c^t}$, where a, b and c are lognormal random variables and q, r, s and t are constants. It is clear that d will be lognormally distributed with median $\beta_d = q\frac{\beta_a^r \beta_b^s}{\beta_c^t}$, and standard deviation $\sigma_d^2 = r^2\sigma_a^2 + s^2\sigma_b^2 + t^2\sigma_c^2$.

In the case of interval imprecision in the distribution parameters of a, b and c we can obtain

$$\bar{\beta}_d = q\frac{\bar{\beta}_a^r \bar{\beta}_b^s}{\underline{\beta}_c^t}, \tag{15}$$

and

$$\underline{\beta}_d = q\frac{\underline{\beta}_a^r \underline{\beta}_b^s}{\bar{\beta}_c^t}, \tag{16}$$

by using the endpoint formulae for interval multiplication[22] with knowledge of the support of the distribution parameters. The logarithmic standard deviation can be obtained from

$$\bar{\sigma}_d^2 = r^2\bar{\sigma}_a^2 + s^2\bar{\sigma}_b^2 + t^2\bar{\sigma}_c^2, \tag{17}$$

and

$$\underline{\sigma}_d^2 = r^2\underline{\sigma}_a^2 + s^2\underline{\sigma}_b^2 + t^2\underline{\sigma}_c^2. \tag{18}$$

This is principally of use when computing the response factor, F, which can be expressed as the product of a number of response factors applying to different pieces of equipment and processes (for example damping effects or modelling effects), i.e. $F = \prod_i F_i$. The F_i are modelled as lognormal random variables and may have interval imprecision in the median [14].

System	Load Probability Density Function	Strength Probability Density Function	Bounds on Failure Probability, P_f
Parallel	Lognormal, with median β_l and standard deviation σ_l	Lognormal, with median $\beta_{ai} \in [\underline{\beta}_{ai}, \bar{\beta}_{ai}]$ and standard deviation $\sigma_{ai} \in [\underline{\sigma}_{ai}, \bar{\sigma}_{ai}]$	$\underline{P}_f = \sum_{c_i \subset S} \min(\phi\left(-\frac{\log \bar{\beta}_{ai} - \log \beta_l}{\sqrt{\bar{\sigma}_{ai}^2 + \sigma_l^2}}\right), \phi\left(-\frac{\log \bar{\beta}_{ai} - \log \beta_l}{\sqrt{\underline{\sigma}_{ai}^2 + \sigma_l^2}}\right)) - (n-1)$ $\bar{P}_f = \min_{c_i \subset S}(\max(\phi\left(-\frac{\log \underline{\beta}_{ai} - \log \beta_l}{\sqrt{\bar{\sigma}_{ai}^2 + \sigma_l^2}}\right), \phi\left(-\frac{\log \underline{\beta}_{ai} - \log \beta_l}{\sqrt{\underline{\sigma}_{ai}^2 + \sigma_l^2}}\right)))$ (19)
Series	Lognormal, with median β_l and standard deviation σ_l	Lognormal, with median $\beta_{ai} \in [\underline{\beta}_{ai}, \bar{\beta}_{ai}]$ and standard deviation $\sigma_{ai} \in [\underline{\sigma}_{ai}, \bar{\sigma}_{ai}]$	$\bar{P}_f = \sum_{c_i \subset S} \max(\phi\left(-\frac{\log \underline{\beta}_{ai} - \log \beta_l}{\sqrt{\bar{\sigma}_{ai}^2 + \sigma_l^2}}\right), \phi\left(-\frac{\log \underline{\beta}_{ai} - \log \beta_l}{\sqrt{\underline{\sigma}_{ai}^2 + \sigma_l^2}}\right))$ $\underline{P}_f = \max_{c_i \subset S}(\min(\phi\left(-\frac{\log \bar{\beta}_{ai} - \log \beta_l}{\sqrt{\bar{\sigma}_{ai}^2 + \sigma_l^2}}\right), \phi\left(-\frac{\log \bar{\beta}_{ai} - \log \beta_l}{\sqrt{\underline{\sigma}_{ai}^2 + \sigma_l^2}}\right)))$ (20)
Single Component	Exponential, with $k_1 \in [\underline{k}_1, \bar{k}_1]$ and $K_H \in [\underline{K}_H, \bar{K}_H]$	Lognormal, with median $\beta \in [\underline{\beta}, \bar{\beta}]$ and standard deviation $\sigma \in [\underline{\sigma}, \bar{\sigma}]$	$\bar{P}_f = \bar{k}_1 \max(\underline{\beta}^{-\underline{K}_H} \exp\frac{(\underline{K}_H \bar{\sigma})^2}{2}, \underline{\beta}^{-\bar{K}_H} \exp\frac{(\bar{K}_H \bar{\sigma})^2}{2})$ $\underline{P}_f = \underline{k}_1 \bar{\beta}^{-\bar{K}_H} \exp\frac{(\underline{K}_H \underline{\sigma})^2}{2}$ (21)
Series	Lognormal, with median $\beta_l \in [\underline{\beta}_l, \bar{\beta}_l]$ and standard deviation $\sigma_l \in [\underline{\sigma}_l, \bar{\sigma}_l]$	Lognormal, with median $\beta_{ai} \in [\underline{\beta}_{ai}, \bar{\beta}_{ai}]$ and standard deviation $\sigma_{ai} \in [\underline{\sigma}_{ai}, \bar{\sigma}_{ai}]$	$\bar{P}_f = \sum_{c_i \subset S} \max(\phi\left(-\frac{\log \underline{\beta}_{ai} - \log \bar{\beta}_l}{\sqrt{\bar{\sigma}_{ai}^2 + \bar{\sigma}_l^2}}\right), \phi\left(-\frac{\log \underline{\beta}_{ai} - \log \bar{\beta}_l}{\sqrt{\underline{\sigma}_{ai}^2 + \underline{\sigma}_l^2}}\right))$ $\underline{P}_f = \max_{c_i \subset S}(\min(\phi\left(-\frac{\log \bar{\beta}_{ai} - \log \beta_l}{\sqrt{\bar{\sigma}_{ai}^2 + \bar{\sigma}_l^2}}\right), \phi\left(-\frac{\log \bar{\beta}_{ai} - \log \beta_l}{\sqrt{\underline{\sigma}_{ai}^2 + \underline{\sigma}_l^2}}\right)))$ (22)

Table 1 – Summary of Failure Probability expressions developed in Section 3.

4. Simple Example - Concrete Containment

4.1 Problem Statement

To demonstrate the results described in the previous sections we will consider a modified version of an example given in [23] with interval imprecision in the coefficient of variation of the random variables. The random variables will be modelled with lognormal distributions because the physical quantities they represent must always be positive. The problem description will be briefly replicated in this section for clarity.

A concrete containment is a structure designed to prevent radioactive release from nuclear power plants to the environment. It is therefore important that the reliability of this structure can be determined accurately, as failing to do so could have severe consequences for the environment and the general public. During the process of determining the reliability of a containment, engineers wish to determine the relationship between applied pressure and failure probability of the containment. A simplified performance function is used to perform reliability analysis without having to run simulations on a complex finite element model. This approach is advantageous as the computational time required is significantly reduced. The approach assumes that the system will fail if the load is larger than the strength.

The containment's strength is considered to be divided between 7 failure mode contributors, all of which may cause system failure. Therefore, this example can be treated as a system composed of 7 components (which are modelled as random variables), connected in series.

The probability of failure for the containment is given by

$$P_f = \int_{S_t < L_t} f(X) dX, \tag{23}$$

where $f(X)$ is the joint probability distribution function of the random variables, $X = (x_1, x_2 ...)$ and S_t and L_t represent the strength and load terms respectively.

The input parameter values assumed in this analysis were taken approximately from the original example [23], but modified to fit lognormal variables and include some imprecision, and are shown in Table 2. The pressure load inside the containment, for the specific accident being considered, was taken to be lognormally distributed with mean 0.575 MPa and standard deviation of 0.117 MPa ($\mu = -0.5737$ MPa and $\sigma = 0.2014$ MPa).

Failure Mode	Logarithmic Mean, μ, e^{β} /MPa	Logarithmic Standard Deviation, σ /MPa
Liner tear around personnel airlock	-0.0943	$[0, 0.0017]$
Basemat shear	-0.0141	$[0, 0.0016]$
Cylinder hoop membrane	0.0853	$[0, 8.8641 \times 10^{-4}]$
Wall-basemat junction shear	0.1231	$[0, 0.0014]$
Cylinder meridional membrane	0.2159	$[0, 8.3320 \times 10^{-4}]$
Dome membrane	0.5911	$[0, 5.345 \times 10^{-4}]$
Personnel air lock door buckling	0.2159	$[0, 0.0013]$

Table 2 – Input parameters for reliability analysis.

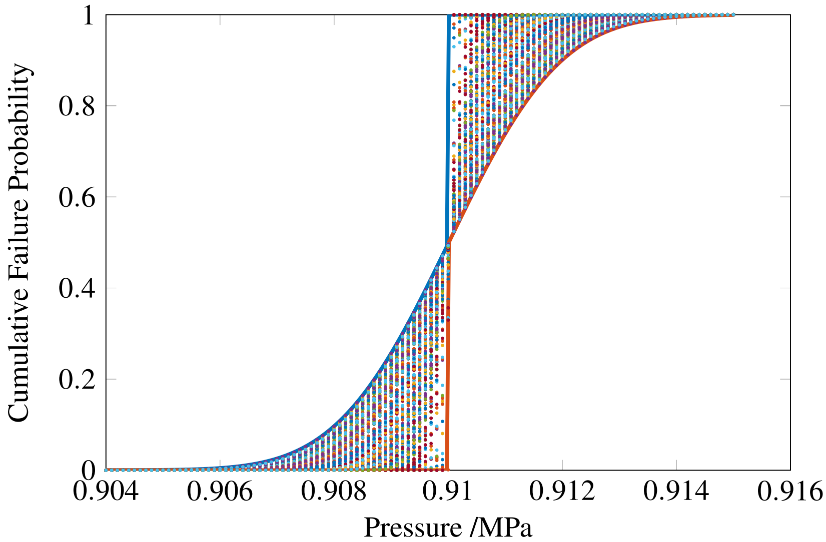

Figure 3 – Probability box representing the fragility curve of the series system, computed analytically. For comparison, the results of a double loop Monte Carlo simulation are shown, which was computed by making 100 epistemic samples.

4.2 Results

The fragility of the series system was bounded using Eqn. 9, and this is plotted in Figure 3, and compared to the empirical CDFs obtained by randomly sampling the epistemic uncertainty.
The failure probability was calculated using Eqn. 20, since the dependence between failure modes was unknown. This revealed that the failure probability was between 0.0086 and 0.0123, which contains the precise probability of failure ($P_f = 0.0122$) given in [23]. This result can be verified by use of double loop Monte Carlo simulation.
These results reveal a good agreement with the expensive simulation procedures in a fraction of the time. In practical cases it would also be necessary to consider uncertainty in the Logarithmic Mean of the random variables which can be easily accounted for given the developments in Section 3..

5. Conclusion

In this paper we have demonstrated methods to analytically propagate probability boxes in commonly used probabilistic safety analysis equations. These equations include series and parallel systems with lognormal fragility distributions and equations where lognormally distributed factors are multiplied. In addition, exponential load distributions are considered. Crucially, we use intervals to model epistemic uncertainty in the parameters of these distributions. This enables the robust quantification of epistemic uncertainty when performing probabilistic safety analysis, particularly in an industrial context. These distributions are sufficient for the analysis of many industrial problems, but in general the imprecise probability methods used could be generalised to other distributions.
These expressions are imprecise probabilistic analogues to many of the probabilistic formulae proposed in Kennedy's paper [11], which have become standard expressions used in probabilistic

safety analysis. These expressions enable engineers to easily conduct probabilistic safety analysis for series and parallel systems under the influence of probability box fragility distributions, without complex simulation techniques.
The work is limited by the fact that only independence of components or complete lack of information on dependence is assumed. In fact, the framework of imprecise probability enables the consideration of correlations between events, for example via convex sets or copula functions. This would be a useful future generalisation of the work in this paper, as any additional information available will allow the bounds on the probability of failure to be tightened.

Acknowledgements

This research has been generously supported by the *EPSRC Centre for Doctoral Training in Nuclear Fission - Next Generation Nuclear* (Grant reference: EP/L015390/1) which is gratefully acknowledged by the authors. The authors would like to express their gratitude for insightful discussion regarding the industrial application of probabilistic safety analysis with Professor Nawal K. Prinja (Technology Director of Clean Energy, Wood plc).

References

[1] M. Modarres and I. S. Kim, "Deterministic and probabilistic safety analysis," in *Handbook of Nuclear Engineering*, D. G. Cacuci, Ed. Boston, MA: Springer US, 2010, pp. 1739–1812. DOI: 10.1007/978-0-387-98149-9_15.

[2] C. N. S. Commission *et al.*, *Probabilistic Safety Assessment (PSA) for Nuclear Power Plants*. Canadian Nuclear Safety Commission, 2005.

[3] D. R. Karanki, H. S. Kushwaha, A. K. Verma, and S. Ajit, "Uncertainty analysis based on probability bounds (p-box) approach in probabilistic safety assessment," *Risk Analysis*, vol. 29, no. 5, pp. 662–675, May 2009. DOI: 10.1111/j.1539-6924.2009.01221.x.

[4] E. Zio and N. Pedroni, "An optimized line sampling method for the estimation of the failure probability of nuclear passive systems," *Reliability Engineering & System Safety*, 19th European Safety and Reliability Conference, vol. 95, no. 12, pp. 1300–1313, Dec. 2010. DOI: 10.1016/j.ress.2010.06.007.

[5] S. Ferson, V. Kreinovich, L. Ginzburg, D. S. Myers, and K. Sentz, "Constructing probability boxes and dempster-shafer structures," Technical report, Sandia National Laboratories, Tech. Rep., 2003. DOI: 10.2172/809606.

[6] E. Patelli, M. Broggi, S. Tolo, and J. Sadeghi, "Cossan software: A multidisciplinary and collaborative software for uncertainty quantification," in *Proceedings of the 2nd ECCOMAS thematic conference on uncertainty quantification in computational sciences and engineering, UNCECOMP 2017, Rhodes Island, Greece*, 2017. DOI: 10.7712/120217.5364.16982.

[7] E. Patelli, D. A. Alvarez, M. Broggi, and M. de Angelis, "Uncertainty management in multidisciplinary design of critical safety systems," *Journal of Aerospace Information Systems*, vol. 12, pp. 140–169, 2015. DOI: 10.2514/1.I010273.

[8] T. D. Le Duy, D. Vasseur, L. Dieulle, C. Bérenguer, and M. Couplet, "Representation of parameter uncertainty with evidence theory in probabilistic risk assessment," in *Proceeding of the Workshop on the Theory of Belief Functions, Brest, France*, 2010.

[9] Z. Qiu, D. Yang, and I. Elishakoff, "Probabilistic interval reliability of structural systems," *International Journal of Solids and Structures*, vol. 45, no. 10, pp. 2850–2860, 2008. DOI: `10.1016/j.ijsolstr.2008.01.005`.

[10] R. J. Budnitz, P. J. Amico, C. A. Cornell, W. J. Hall, R. P. Kennedy, J. W. Reed, and M. Shinozuka, "Approach to the quantification of seismic margins in nuclear power plants," Lawrence Livermore National Lab., CA (USA), NUREG/CR-4334; UCID-20444, Aug. 1, 1985.

[11] R. P. Kennedy, C. Cornell, R. Campbell, S Kaplan, and H. Perla, "Probabilistic seismic safety study of an existing nuclear power plant," *Nuclear Engineering and Design*, vol. 59, no. 2, pp. 315–338, 1980. DOI: `10.1016/0029-5493(80)90203-4`.

[12] N. K. Prinja, A. Ogunbadejo, J. Sadeghi, and E. Patelli, "Structural reliability of pre-stressed concrete containments," *Nuclear Engineering and Design*, DOI: `10.1016/j.nucengdes.2016.11.036`.

[13] H.-l. Sun and W.-x. Yao, "The basic properties of some typical systems' reliability in interval form," *Structural Safety*, vol. 30, no. 4, pp. 364–373, 2008. DOI: `10.1016/j.strusafe.2007.05.003`.

[14] C. R. Sundararajan, *Probabilistic structural mechanics handbook: theory and industrial applications*. Springer Science & Business Media, 2012.

[15] J. Collet, "Some remarks on rare-event approximation," *IEEE Transactions on Reliability*, vol. 45, no. 1, pp. 106–108, DOI: `10.1109/24.488924`.

[16] L. Rüschendorf, "Fréchet-bounds and their applications," *Advances in probability distributions with given marginals*, pp. 151–187, 1991. DOI: `10.1007/978-94-011-3466-8_9`.

[17] J. H. Kim, I.-K. Choi, and J.-H. Park, "Seismic evaluation by using composite standard deviation," in *Transactions of the Korean Nuclear Society Autumn Meeting*, Jeju, Korea, Oct. 2010.

[18] S. Kaplan, V. M. Bier, and D. C. Bley, "A note on families of fragility curves - is the composite curve equivalent to the mean curve?" *Reliability Engineering & System Safety*, vol. 43, no. 3, pp. 257–261, 1994.

[19] K. Kapur and L. Lamberson, *Reliability in engineering design*. Wiley, 1977.

[20] J. Braverman, J Xu, B. Ellingwood, C. Costantino, R. Morante, and C. Hofmayer, "Evaluation of the seismic design criteria in asce/sei standard 43-05 for application to nuclear power plants," *US Nuclear Regulatory Commission, Job Code N*, vol. 6112, pp. 1–73, 2007.

[21] R. P. Kennedy, "Performance-goal based (risk informed) approach for establishing the sse site specific response spectrum for future nuclear power plants," *Nuclear Engineering and Design*, vol. 241, no. 3, pp. 648–656, 2011. DOI: `10.1016/j.nucengdes.2010.08.001`.

[22] R. E. Moore, R. B. Kearfott, and M. J. Cloud, *Introduction to interval analysis*. Philadelphia, PA: Society for Industrial and Applied Mathematics, 2009, 223 pp. DOI: `10.1137/1.9780898717716.bm`.

[23] M. Modarres, M. P. Kaminskiy, and V. Krivtsov, *Reliability engineering and risk analysis: a practical guide*. CRC press, 2016, ch. 6, pp. 281–282. DOI: 10.1016/S0951-8320(02)00008-X.